THE POTENTIAL SYSTEMATIC BEHAVIOUR OF SOME PANEL SURVEY ESTIMATES

W. H. Williams and C. L. Mallows, Bell Telephone Laboratories, Incorporated

ABSTRACT

Panel surveys involved repeated observations made on the same sample of population units. In some of these studies systematic biases have appeared; specifically, if R_i is an estimate made from panels appearing in the sample for the i^{th} time, then it has been observed that these estimates sometimes vary systematically with i. It is tempting to interpret these phenomena as being due to conditioning of the panel of respondents.

In this paper it is shown that these systematic changes can be a result of the characteristics of the response probabilities. This is true whether the estimates are computed only for those individuals who are identical from one interview time to the next, or on all available persons, or on those persons appearing only once.

A discussion of similar results for complete followup surveys, and the description of a simple "adjusted" estimation procedure are also included in the paper. Some numerical examples are presented to show that in some very innocuout situations the potential biases can be very large.

I. INTRODUCTION

Panel surveys involve the periodic observation of the same population units. In some such studies, systematic biases have appeared which do not appear to have been fully explained. For example, in the Current Population Survey conducted by the U.S. Bureau of the Census [7], the unemployment rate of those persons interviewed for the first time appears to be considerably higher than on subsequent interviews, see Waksberg and Pearl [8] and the Report of the President's Commitee to Appraise Employment and Unemployment Statistics [9]. One of the main purposes of this paper is to show that, under certain simple hypotheses each of several natural estimates made at two different interview times, may exhibit systematic changes simply as a result of sampling characteristics.

Panel surveys have many variations. Sometimes a selected sample is followed up as completely as possible at each of the subsequent observation times. On the other hand rotation designs involve the month-to-month (say) retention of some sampling units and the replacement of others. These later designs can be used to produce good estimates at specific points in time as well as good estimates of change. The details of rotation sampling and panel studies in general will not be described in this paper because there is a large literature on the subject, see for example Hansen, Hurwitz and Madow [1], Cochran [2], Patterson [3], Eckler [4], Rao and Graham [5], and Kish [6].

Generally, if R_i is the estimate made from those rotation groups which are appearing in the sample for the i^{th} time, then it has been sometimes observed that there are significant and systematic changes in these estimates when considered as a function of i. A study of this problem by one of the authors lead to a possible explanation of these phenomena; see Williams [10]. In that paper it was shown that if, (1) the probability of nonresponse for a selected unit is monotonically related to the characteristic under study, and if (2) this same probability changes from one observation time to the next, then systematic changes in the expectation of the estimator must occur.

These two hypotheses are very reasonable ones. It is well known that nonresponse is sometimes related to the characteristic being measured. For example, families with no children are much more likely to be missed than families with children. There is a large literature on this type of behavior, see for example Kish [6]. Also there can be little doubt that the probabilities change from one interview time to the next. This is often clearly revealed by a changing nonresponse rate, which can change only if the nonresponse probabilities change; and after all it is the continuing goal of every survey manager to achieve a higher response rate.

The question remains, however, as to whether any of the observed systematic changes could be caused by other factors. For example, is it true that people tend to answer questions about their economic status differently in a first interview than in a second? To examine this question it is tempting to construct estimates which are based only upon those individuals who appear in the survey both at T_1 and T_2. The argument is that systematic changes in the estimate for this matched set must be the result of factors other than purely statistical ones. This conclusion is false. The reason is that systematic changes in the probability of nonresponse may be ensuring that the matched set of individuals is not representative.

The same difficulty can arise in any study in which the same individuals are observed at repeated intervals and in which nonresponse occurs. It can occur for example in the so-called complete follow-up surveys so frequently used to examine socioeconomic and medical characteristics. It is true that such studies give a detailed account of the history of the selected individuals, and that changes in the observation from T_1 to T_2 are certainly due to real changes in these individuals or at least to the response given by them. As we shall show, however, it is not necessarily appropriate to estimate changes in the population from the available observations on identical persons.

Also in this paper we undertake a comparison of estimates based on identical persons, on all persons, and on those persons interviewed only at one observation period. Since these comparisons appear to be awkward algebraically some numerical results are included.

Finally a simple "adjusted" estimation scheme is described. It is not recommended unreservedly however because it can exhibit poor behavior. The authors hope to report on additional work on the estimation problem in a future paper.

Since it was examination of the problem of systematic bias in unemployment statistics that led to the work described in this paper, we have chosen to describe the analysis in terms of employment and unemployment. However it is important to notice that none of the data in this paper are real and that the relationship of these models to the real problem has yet to be examined.

II. THE STATISTICAL MODEL

We have chosen to use a simple model for illustration. The increased algebra required for more general cases would detract from the presentation sufficiently that it seems better to avoid it. Consequently a two catagory model, employed and unemployed, is considered.

Suppose that a sample design results in the selection of a certain geographical area and that the employment status of all persons living in that area is to be determined both at time T_1 and again at a later time, T_2. For simplicity, we neglect the effect of population mobility. Many sample designs will specify a subsampling of persons in the selected area; the remarks we shall make apply to these cases also but for simplicity they are not discussed explicitly. Similarly, the higher structure of the sample design is also ignored with no loss in generality. Given these assumptions, all persons in the area to be sampled can be classified in the manner described in Table 2.1.

		Employment Status at Time T_2	
		Unemployed	Employed
Employment Status at Time T_1	Unemployed	N_{uu}	N_{ue}
	Employed	N_{eu}	N_{ee}

Table 2.1: Numbers of Persons Employed and Unemployed.

Using the notation of Table 2.1, the true unemployed/employed ratio at T_1 is given by

$$R_1 = \frac{N_{uu}+N_{ue}}{N_{eu}+N_{ee}} , \qquad (2.1)$$

and at T_2 by

$$R_2 = \frac{N_{uu}+N_{eu}}{N_{ue}+N_{ee}} . \qquad (2.2)$$

Algebraically, it can be seen that $R_1 = R_2$ if and only if $N_{ue} = N_{eu}$, that is, when the number of persons who have found employment in the period from T_1 to T_2 is equal to the number of persons who have lost it. Intuitively, this is an obvious condition for the unemployment ratio to remain constant.

When the actual sampling of the selected area begins, the classification of Table 2.1 will not be adequate, since not all persons will be interviewed. Consequently, each individual can be classified as employed, unemployed, or not interviewed at each of T_1 and T_2, as in Table 2.2. After the second round of interviews, the nine frequencies in this table will be known. We are assuming here that those persons who are not interviewed at either T_1 or T_2 can still be counted, so that the frequency F_{oo} is known. In some kinds of survey F_{oo} may remain unknown, or a rough estimate of it may become available. This has no effect on the subsequent discussion in this paper, which is directed towards estimation of the unemployed/employed ratios using the other eight frequencies, and which largely ignores sampling variations in the observed frequencies.

		Status at Time T_2		
		Unemployed	Employed	Not Interviewed
Status at Time T_1	Unemployed	F_{uu}	F_{ue}	F_{uo}
	Employed	F_{eu}	F_{ee}	F_{eo}
	Not Interviewed	F_{ou}	F_{oe}	F_{oo}

Table 2.2: Observed Numbers of Persons in Various Categories

To construct an elementary model, let

P_u = the probability that an unemployed person actually appears in the sample at T_1;

P_e = the probability that an unemployed person actually appears in the sample at T_1;

P_{uu} = the probability that an individual is interviewed at T_2 given that he was interviewed at T_1 and was unemployed at both T_1 and T_2;

P_{ue} = the probability that an individual is interviewed at T_2 given that he was interviewed at T_1 and was unemployed at T_1 and employed at T_2;

P_{eu} = the probability that an individual is interviewed at T_2 given that he was interviewed at T_1 and was employed at T_1 and unemployed at T_2;

P_{ee} = the probability that an individual is interviewed at T_2 given that he was interviewed at T_1 and was employed at both T_1 and T_2.

Finally, let Q_{uu}, Q_{ue}, Q_{eu} and Q_{ee} represent probabilities similar to P_{uu}, P_{ue}, P_{eu} and P_{ee} except that the Q's are conditional to the individual not being interviewed at T_1. For example Q_{uu} = the probability that an individual is interviewed at T_2 given that he was not interviewed at T_1 and was employed at both T_1 and T_2.

Ideally each of those probabilities would equal unity because all persons in the selected area are theoretically to be included in the sample. However nonresponse problems will virtually always ensure that these probabilities are not unity. Consequently it is interesting to construct a table of expected sample numbers based on the given three-by-three classification. These expectations are displayed in Table 2.3.

III. THE STUDY OF IDENTICAL PERSONS

The array of observed sample numbers Table 2.2, can be used to construct an estimator based only on individuals who

		STATUS AT TIME T_2		
		UNEMPLOYED	EMPLOYED	NOT INTERVIEWED
STATUS AT TIME T_1	UNEMPLOYED	$N_{uu}P_uP_{uu}$	$N_{ue}P_uP_{ue}$	$N_{uu}P_u(1-P_{uu}) + N_{ue}P_u(1-P_{ue})$
	EMPLOYED	$N_{eu}P_eP_{eu}$	$N_{ee}P_eP_{ee}$	$N_{eu}P_e(1-P_{eu}) + N_{ee}P_e(1-P_{ee})$
	NOT INTERVIEWED	$N_{uu}(1-P_u)Q_{uu}$ $+N_{eu}(1-P_e)Q_{eu}$	$N_{ue}(1-P_u)Q_{ue}$ $+N_{ee}(1-P_e)Q_{ee}$	$N_{uu}(1-P_u)(1-Q_{uu}) + N_{ue}(1-P_u)(1-Q_{ue})$ $+N_{eu}(1-P_e)(1-Q_{eu}) + N_{ee}(1-P_e)(1-Q_{ee})$

Table 2.3: Expected Sample Numbers

are observed at both T_1 and T_2. Using the table as a guide, it can be seen that the number of persons who are unemployed at T_1 and who are interviewed again at T_2 (i.e., in either the employed or unemployed category at T_2) is given by $F_{uu} + F_{ue}$. Similarly, the number of persons employed at T_1 and interviewed at T_2 is given by $F_{eu} + F_{ee}$. Consequently, the unemployment ratio at T_1, based only on those individuals who appear both at T_1 and T_2 is given by,

$$\hat{R}_1 = \frac{F_{uu}+F_{ue}}{F_{eu}+F_{ee}} . \qquad (3.1)$$

Similarly, the unemployment ratio at time T_2 for this same group of identical individuals is given by,

$$\hat{R}_2 = \frac{F_{uu}+F_{eu}}{F_{ue}+F_{ee}} . \qquad (3.2)$$

Furthermore, it can be seen that the difference $\hat{R}_1 - \hat{R}_2$ is given by,

$$\hat{R}_1 - \hat{R}_2 = (F_{ue}-F_{eu})(\text{positive factor}), \qquad (3.3)$$

so that $\hat{R}_1 = \hat{R}_2$ if and only if

$$F_{eu} = F_{eu}. \qquad (3.4)$$

In the usual survey situation, these frequencies will be large, so that under the model of Section II and neglecting sampling variability, (3.4) becomes

$$N_{ue}P_uP_{ue} = N_{eu}P_eP_{eu}. \qquad (3.4a)$$

In the case in which there is no overall change in employment, (i.e., $N_{ue} = N_{eu}$), this equation can be written as,

$$\frac{P_u}{P_e} = \frac{P_{eu}}{P_{ue}} . \qquad (3.5)$$

The result of Eq. (3.5) means that there will be a change in the observed unemployment ratio unless the ratio of the probability of interviewing an unemployed person to the probability of interviewing an employed person at, time T_1 is the same as the ratio of the corresponding probabilities at time T_2 for persons who were observed at T_1 and whose employment status has changed between T_1 to T_2. This change in the observed unemployment ratio will occur even though there is no change in the true unemployment ratio.

To explain this in a simpler case, consider the model in which the probabilities at T_2 are independent of the status at T_1 and of whether the individual is observed at T_1, as follows: $P_{uu} = P_{eu} = P_{2u}$, $P_{ue} = P_{ee} = P_{2e}$. Also we now write $P_u = P_{1u}$ and $P_e = P_{1e}$. Then Eq. (3.4a) can be written as,

$$N_{ue}P_{1u}P_{2e} = N_{eu}P_{1e}P_{2u}, \qquad (3.6)$$

and Eq. (3.5) as,

$$\frac{P_{1u}}{P_{1e}} = \frac{P_{2u}}{P_{2e}} . \qquad (3.7)$$

The interpretation of Eq. (3.7) is that the ratio of the probability of interviewing an unemployed person to the probability of interviewing an employed person must be the same at T_1 and T_2, otherwise there will be a change in the expected unemployment ratio from T_1 to T_2. This is true even though there has been no change in the true unemployment ratio and the estimates are based on identical individuals. Is Eq. (3.7) likely to hold in practice? It has been reported by Williams [10] that some practitioners feel that $P_{1u} > P_{1e}$. Consequently, if a survey manager subsequently reduces the nonresponse so effectively that P_{2u} and P_{2e} are approximately unity then the difference $\hat{R}_1 - \hat{R}_2$ must be positive and a systemmatic change in the estimator based on the identical persons will be observed. A similar result was also found by Williams [10] for estimators which are based on the complete sample at each of T_1 and T_2.

IV. A COMPARISON OF THE ESTIMATES BASED ON IDENTICAL, UNMATCHED, AND ALL PERSONS

In the previous section an analysis was made of the estimates based only on those persons appearing in the sample both at T_1 and T_2. Estimates can also be obtained which are based on those individuals who appear only once, either at T_1 and T_2. So altogether three pairs of estimates of R_1 and R_2 might be in hand; one based on identical persons, one on "single" persons and one on all persons. Consequently, it is interesting to ask whether all three of these estimates must necessarily exhibit a systematic change? It is possible for one estimate of the difference, $R_1 - R_2$, to behave differently from the others? And what are the magnitudes of the possible biases in the various estimates of $R_1 - R_2$?

The estimates based on identical individuals have already been described in

Section III. They were formed by simply picking out the appropriate sample numbers from Table 2.2. The estimate based on "single" individuals and the estimate based on all available persons can easily be formed in the same way. For example, the estimates based on the total number of persons, and their values under our model in the simple case of independence, are given by

$$\hat{R}_{1t} = \frac{F_{uu}+F_{ue}+F_{uo}}{F_{eu}+F_{ee}+F_{eo}} \approx \left(\frac{N_{uu}+N_{ue}}{N_{eu}+N_{ee}}\right) \frac{P_{1u}}{P_{1e}} , \tag{4.1}$$

and

$$\hat{R}_{2t} = \frac{F_{uu}+F_{eu}+F_{ou}}{F_{ue}+F_{ee}+F_{oe}} \approx \left(\frac{N_{uu}+N_{eu}}{N_{eu}+N_{ee}}\right) \frac{P_{2u}}{P_{2e}} . \tag{4.2}$$

The estimates based on the individuals who appear only on a single occasion are, (in the simple case of independence)

$$\hat{R}_{1s} = \frac{F_{ou}}{F_{eo}} \approx \frac{N_{uu}(1-P_{2u}) + N_{eu}(1-P_{2e})}{N_{eu}(1-P_{2u}) + N_{ee}(1-P_{2e})} \left(\frac{P_{1u}}{P_{1e}}\right) \tag{4.3}$$

$$\hat{R}_{2s} = \frac{F_{ou}}{F_{oe}} \approx \frac{N_{uu}(1-P_{1u}) + N_{eu}(1-P_{1e})}{N_{ue}(1-P_{1u}) + N_{ee}(1-P_{1e})} \left(\frac{P_{2u}}{P_{2e}}\right) \tag{4.4}$$

Unfortunately, the differences $\hat{R}_1 - \hat{R}_2$, $\hat{R}_{1t} - \hat{R}_{2t}$, and $\hat{R}_{1s} - \hat{R}_{2s}$ do not appear to have algebraic forms which facilitate easy comparisons. Consequently, a numerical study was undertaken. A large number of cases were computed; six are presented in the appendix. Each of the examples given there is arrayed in the same way, with the relevant population parameters followed by two tables. In the first table the expected sample numbers corresponding to Table 2.3 are laid out. Below this table, $E(n_1)$ and $E(n_2)$, the expected responses at T_1 and T_2 are listed. Table 2 contains the estimates of R_1, R_2, and $R_1 - R_2$ based on each of (i) identical persons, (ii) all persons (total) (iii) persons interviewed only at T_1 or T_2, (single) (iv) an adjusted estimator which will be described below.

Examination of the numerical results enables one to make certain comments:

1. Substantial biases appear with apparently innocous probability differences and with very low nonresponse rates. In example A, the response rate at T_1 is over 89 percent and at T_2, it is approaching 95 percent. These are response rates which are characteristic of some of the surveys run by the U.S. Bureau of the Census but are not matched consistently by any survey group. Nevertheless, the estimates based on all sampled persons suggest a change of about 14 percent in the unemployed/employed ratio. Keep in mind that $N_{eu} = N_{eu}$, so that there is no real change in unemployment, and also that sampling variability is being ignored. Even the estimate based on an identical set of individuals has a bias of about five percent. The estimates of R_1, R_2 and $R_1 - R_2$ based on the "singles" are so bad that it is disturbing to consider the possibility of their use.

Example B has been included to further illustrate the major effect that these probability differences can have on the estimates. The only difference between Example A and Example B is that the second stage response probabilities for unemployed persons, i.e., $P_{uu}, Q_{uu}, P_{eu}, Q_{eu}$ have dropped substantially. It is tempting to react to this case by taking the attitude that these probabilities are unrealistically low. But are they? And how would one know it, because the response rate at T_2 has dropped only one and one-half percent, which is very little different from Example A and in general is still a very high response rate. The important point however is that with almost no warning the biases in R_1, R_2 and $R_1 - R_2$ have gone from bad, in Example A to disastrous in Example B. The "total" change estimate now has a bias of over 55 percent! The estimates of R_1 and R_2 based on the identicals are clearly very bad, but in addition the "identical" estimate of $R_1 - R_2$ is now about 14 percent, or three times as big as it was in Example A.

2. Examples A and B have the same amount of shifting from one category to the other, i.e., $N_{ue} = N_{eu} = 100$ in both cases. This shifting does affect the biases. For example, suppose the probabilities of case A are used with the populations given below:

	N_{uu}	N_{ue}	N_{eu}	N_{ee}
i	400	00	00	9600
ii	300	100	100	9500
iii	200	200	200	9400
iv	100	300	300	9300
v	0	400	400	9200

These five examples are constructed so that the true unemployment ratio remains constant at 4.17%. It will be found that the biases in both the "total" and "identical" estimates get worse as the shifting around increases. On the other hand, the singles estimates get better. The latter are still not satisfactory but at least do not exhibit the extremely bad behavior of Examples A and B.

3. Calculation of case v in item 2 above reveals the distressing fact that the three estimates of the change $R_1 - R_2$ from identicals, total, and singles do not even necessarily have the same sign.

4. The Q probabilities do not seem to play a major role in the biases. These probabilities cannot affect the "identical" estimates at all and have only a small effect on the bias in the "singles" estimate. The estimates based on total persons are affected even more slightly because these are a combination of the single and the matched.

5. An examination of the tables of expected numbers reveals characteristics which are likely to be quite perplexing. For example the number of interviewed, employed, persons (at T_2), who were not interviewed at T_1 may be two or three times the number of persons who were employed at T_1 and not interviewed at T_2.

Specifically, in case A, the number of persons interviewed at T_2 and found to be employed, but who were not interviewed at T_1 is 999. This means that 999 employed persons who were not interviewed at T_1 were found in the sample area at T_2. On the other hand only 437 employed persons were interviewed at T_1 and lost to the sample at T_2. Thus it appears that one is "finding" far more employed persons at T_2 than were lost at T_1. This result might be interpreted as showing mobility of the population, so that unemployed persons move and show up somewhere else as an employed persons. Example A shows that this mobility feature may well be only a reflection of the response probability structure. This is not to say that the real population does not have such a characteristic, this example shows only that it will have to be demonstrated with other evidence.

6. The examples presented in the statistical appendix have what must be considered high expected response rates. If the probabilities are dropped so that the response rates are in the vicinity of 60 to 70 percent, the biases become far worse that those shown in cases A and B. Unfortunately, in practice, more surveys seem to operate in this range than with response rates of 90 percent.

7. Example C is constructed so that there is an even split of the population between the two categories. Again $N_{ue} = N_{eu}$ so that there is no change in the ratio in the two categories from T_1 to T_2. Since unemployment rates of this magnitude occur very seldom, it may be more helpful to discuss this example in terms of persons who say that they are going to vote for candidate U or for candidate E. The probabilities are numerically the same as those used in Example A. Now however they are interpreted as indicating that the voters for one candidate are more easily interviewed that those of the other. Both at T_1 and T_2 there is a response rate of approximately 91 percent which again can be considered to be high. However, the three estimates of $R_1 - R_2$, "identicals", "total" and "singles" give the appearance of a major swing to candidate E. Just how big a swing is determined by which of the three estimates is being considered. Unfortunately, in reality there is no swing to either of the candidates.

8. In practice the eight numbers (apart from the marginal totals) given in Table 1 of each of the examples are available for estimation purposes. Unfortunately our model has fourteen parameters, six P's, four Q's and four N's. In this situation, there are a number of simplifying assumptions that can be made to reduce the fourteen parameters to eight. One has been used in this paper. It consists of assuming that the four Q's are equal to the corresponding P's. In addition the assumptions $P_{uu} = P_{eu} = P_{2u}$ and $P_{ee} = P_{ue} = P_{2e}$ reduces the number of parameters to eight, specifically, two first stage P's, two second stage P's and four N's. Now the expressions for the expected cell numbers can be equated to the realized numbers and the resultant equations solved for the unknown eight parameters. This crude "adjusted" estimation scheme was used in all the examples shown in the appendix. While it behaves better than the other estimators in all of the present cases, and indeed did so in many of the computed examples which have not been included in the paper, it does not always behave so well. The estimation possibilities for these incomplete response cases required further investigation and the authors hope to report on this in a future paper.

V. COMPLETE FOLLOWUP STUDIES

Many sociological and medical surveys involve the repeated interviewing of all of the selected individuals. For example, a medical survey may involve the random selection of a group of people who are then all given periodic medical examinations. New persons may or may not be included in the survey at later stages, but to point out the potential difficulties in a simple way, suppose that no new individuals enter the study and that there is no loss from the original sample. Thus we put $P_{uu} = P_{ue} = P_{eu} = P_{ee} = 1$ and $Q_{uu} = Q_{ue} = P_{eu} = Q_{ee} = 0$ in the earlier models. While this model does not represent realistically the characteristics of sampling for unemployment statistics, for consistency we shall continue to refer to the classifications in that way. Given these assumptions the expectations are as given in Table 5.1.

		Status at Time T_2		
		Unemployed	Employed	Not Interviewed
Status at Time T_1	Unemployed	$N_{uu}P_u$	$N_{ue}P_u$	0
	Employed	$N_{eu}P_e$	$N_{ee}P_e$	0
	Not Interviewed	0	0	0

Table 5.1: Expected Sample Numbers, Complete Followups.

Consequently, at T_1,

$$\hat{R}_1 \approx \left(\frac{N_{uu}+N_{ue}}{N_{eu}+N_{ee}}\right)\frac{P_u}{P_e}\ , \qquad (5.1)$$

and at T_2,

$$\hat{R}_2 \approx \frac{N_{uu}P_u+N_{eu}P_e}{N_{ue}P_u+N_{ee}P_e}\ , \qquad (5.2)$$

Consequently,

$$\hat{R}_1 - \hat{R}_2 = (N_{eu}P_u - N_{eu}P_e)(\text{positive factor}), \qquad (5.3)$$

so that (neglecting sampling variability), $\hat{R}_1 = \hat{R}_2$ if and only if

$$N_{eu}/N_{ue} = P_u/P_e. \qquad (5.4)$$

From this result it can be seen that even when there is no overall change in "unemployment" from T_1 to T_2 (so that $N_{eu} = N_{ue}$), there will be change in the expectation of the estimator unless $P_u = P_e$. This rather disturbing result says that unless the probabilities of response at T_1 are known and are dealt with appropriately then the estimator may change from T_1 to T_2 even though the study involves an identical set of individuals at T_1 and T_2. There is no difficulty extending these results to followup studies with different and more complex sampling schemes.

Examples D, E and F have been included in the appendix to illustrate what may happen in the complete follow up case. Example D assumes that there is no response loss at T_2 and no new persons brought into the survey. The expected response is 89 percent. Even in this ideal case a one percent bias has crept in. In Example E, the second stage P probabilities have been lowered, so that some persons are lost from the survey but no new ones are added, i.e., the Q's remain equal to zero. The response rates are still high: 89 percent at T_1 and over 84 percent at T_2, nevertheless the biases in all of the estimates except the adjusted one have become unacceptably large. The "identical" change bias is about five percent and the "total" change bias is about ten percent. Since there are no persons appearing only at T_2, the singles estimate does not apply.

In Example F, P_{uu} and P_{eu} have been lowered substantially, to 0.50. It is important to notice that in practice there would be extreme difficulty in detecting whether one was more nearly in case E or F because the response rates are very nearly the same. The biases however have changed considerably. In case E, the biases were simply very bad, in case F they are much worse.

It is hoped that these simple examples will help to show the magnitude of the potential danger in panel surveys of this kind.

VI. SUMMARY AND GENERALIZATIONS

The important point made in this paper is that systematic changes will occur in estimates from one stage of a survey to the next if the probabilities of nonresponse are not the same for all of the population categories. Williams [10] showed that this was possible for the overall sample and in this paper the same systematic biases have been shown to be possible even if one computes estimates based on an identical set of individuals.

This paper also has included a discussion of complete followup surveys and a simple "adjusted" estimation scheme. Some numerical examples have been included which show the extreme biases that can easily be encountered in the estimates of change from one observation period of the next. This is true whether the estimate of change is based on identical persons, on all persons or on those persons appearing only for a single interview.

Distortions caused by the incomplete responses can mislead a researcher. For example, it is pointed out that a "mobility" characteristic could very well be a result of these nonresponse problems.

The model presented in this paper is a very simple one. It can be generalized easily to more categories and to additional reinterviews. With some increase in complexity the case of a continuous variate y_i can also be covered. We have chosen not to attempt extensive generalization in this paper. The list of cases that might be worth spelling out in detail is as long as the number of different schemes used in practice. It seems important simply to point out that these awkward biases can and probably do occur in practice.

REFERENCES

1. Hansen, M. H., Hurwitz, W. N. and Madow, W. G., Sample Survey Methods and Theory, John Wiley and Sons, New York, Vols. I and II, 1953.

2. Cochran, W. G., Sampling Techniques, Second Edition, John Wiley and Sons, New York, 1963.

3. Patterson, H. D., Sampling on successive occasions with partial replacement of units, Jour. Roy. Stat. Soc., B, 12, 241-255, 1950.

4. Eckler, A. R., Rotation Sampling. Ann. Math., Stat., 26, 664-458,[1955].

5. Rao, J. N. K., and Graham, Jack E., Rotation designs for sampling on repeated occasions. Jour. Amer. Stat. Assoc. 59, 495-509, 1964.

6. Kish, Leslie, Survey Sampling, John Wiley and Sons, New York, 1965.

7. U.S. Bureau of the Census, The Current Population Survey - A Report on Methodology. Technical Paper No. 7, U.S. Government Printing Office, Washington, D.C., 1963.

8. Waksberg, Joseph, and Pearl, Robert B., The effects of repeated interviews in the current population survey. Paper presented at the 47th National Conference of the American Marketing Association, Dallas, Texas, 1964.

9. Measuring Employment and Unemployment, Report of the President's Committee to Appraise Employment and Unemployment Statistics. The White House, 1962.

10. Williams, W. H., The systematic bias effects of incomplete responses in rotation samples. To appear in the Public Opinion Quarterly.

11. Durbin, J., and Stuart, A., Differences in response rates of experienced and inexperienced interviewers. Jour. Roy. Stat. Soc., 114, 164-206, 1951.

APPENDIX

EXAMPLE A

First Stage Response Probabilities

$P_u = 0.93 \qquad P_e = 0.89$

Second Stage Response Probabilities

$P_{uu} = 0.87 \quad P_{eu} = 0.96 \quad P_{eu} = 0.84 \quad P_{ee} = 0.95$

$Q_{uu} = 0.87 \quad Q_{ue} = 0.96 \quad Q_{eu} = 0.84 \quad Q_{ee} = 0.95$

True Population Figures

$N_{uu} = 300 \quad N_{eu} = 100 \quad N_{eu} = 100 \quad N_{ee} = 9500$

TABLE 1: EXPECTED SAMPLE NUMBERS

	U2	E2	N2	T2
U1	243	89	40	372
E1	75	8032	437	8544
N1	28	999		
T1	345	9121		

$E(n_1) = 8916$

$E(n_2) = 9466$

TABLE 2: ESTIMATES OF UNEMPLOYMENT RATIOS (PERCENT)

	TRUE	IDENTICALS	TOTAL	SINGLES	ADJUSTED
R1	4.17	4.10	4.35	9.15	4.17
R2	4.17	3.91	3.78	2.75	4.11
R1 - R2	0.00	0.19	0.57	6.40	0.05

EXAMPLE B

First Stage Response Probabilities

$P_u = 0.93$ $P_e = 0.89$

Second Stage Response Probabilities

$P_{uu} = 0.50$ $P_{ue} = 0.96$ $P_{eu} = 0.50$ $P_{ee} = 0.95$

$Q_{uu} = 0.40$ $Q_{ue} = 0.96$ $Q_{eu} = 0.40$ $Q_{ee} = 0.95$

True Population Figures

$N_{uu} = 300$ $N_{ue} = 100$ $N_{eu} = 100$ $N_{ee} = 9500$

TABLE 1: EXPECTED SAMPLE NUMBERS

	U2	E2	N2	T2
U1	139	89	143	372
E1	44	8032	467	8544
N1	13	999		
T1	197	9121		

$E(n_1) = 8916$

$E(n_2) = 9318$

TABLE 2: ESTIMATES OF UNEMPLOYMENT RATIOS (PERCENT)

	TRUE	IDENTICALS	TOTAL	SINGLES	ADJUSTED
R1	4.17	2.83	4.35	30.65	4.08
R2	4.17	2.27	2.16	1.28	4.08
R1 - R2	0.00	0.57	2.20	29.37	-0.01

EXAMPLE C

First Stage Response Probabilities

$P_u = 0.93$ $P_e = 0.89$

Second Stage Response Probabilities

$P_{uu} = 0.87$ $P_{ue} = 0.96$ $P_{eu} = 0.84$ $P_{ee} = 0.95$

$Q_{uu} = 0.87$ $Q_{ue} = 0.96$ $Q_{eu} = 0.84$ $Q_{ee} = 0.95$

True Population Figures

$N_{uu} = 4000$ $N_{ue} = 1000$ $N_{eu} = 1000$ $N_{ee} = 4000$

TABLE 1: EXPECTED SAMPLE NUMBERS

	U2	E2	N2	T2
U1	3236	893	521	4650
E1	748	3382	320	4450
N1	336	485		
T1	4320	4760		

$E(n_1) = 9100$

$E(n_2) = 9080$

TABLE 2: ESTIMATES OF UNEMPLOYMENT RATIOS (PERCENT)

	TRUE	IDENTICALS	TOTAL	SINGLES	ADJUSTED
R1	100.00	99.99	104.49	162.55	100.00
R2	100.00	93.20	90.76	69.25	97.63
R1 - R2	00.00	6.79	13.74	93.30	2.37

EXAMPLE D

First Stage Response Probabilities

$P_u = 0.93$ $P_e = 0.89$

Second Stage Response Probabilities

$P_{uu} = 1.00$ $P_{ue} = 1.00$ $P_{eu} = 1.00$ $P_{ee} = 1.00$

$Q_{uu} = 0.00$ $Q_{ue} = 0.00$ $Q_{eu} = 0.00$ $Q_{ee} = 0.00$

True Population Figures

$N_{uu} = 300 \quad N_{ue} = 100 \quad N_{eu} = 100 \quad N_{ee} = 9500$

TABLE 1: EXPECTED SAMPLE NUMBERS

	U2	E2	N2	T2
U1	279	93	0	372
E1	89	8455	0	8544
N1	0	0		
T1	368	8548		

$E(n_1) = 8916$

$E(n_2) = 8916$

TABLE 2: ESTIMATES OF UNEMPLOYMENT RATIOS (PERCENT)

	TRUE	IDENTICALS	TOTAL	SINGLES	ADJUSTED
R1	4.17	4.35	4.35	0.00	4.35
R2	4.17	4.31	4.31		4.31
R1 - R2	0.00	0.05	0.04		0.05

EXAMPLE E

First Stage Response Probabilities

$P_u = 0.93 \qquad P_e = 0.89$

Second Stage Response Probabilities

$P_{uu} = 0.87 \; P_{ue} = 0.96 \; P_{eu} = 0.84 \; P_{ee} = 0.95$

$Q_{uu} = 0.00 \; Q_{ue} = 0.00 \; Q_{eu} = 0.00 \; Q_{ee} = 0.00$

True Population Figures

$N_{uu} = 300 \quad N_{ue} = 100 \quad N_{eu} = 100 \quad N_{ee} = 9500$

TABLE 1: EXPECTED SAMPLE NUMBERS

	U2	E2	N2	T2
U1	243	89	40	372
E1	75	8032	437	8544
N1	0	0		
T1	317	8122		

$E(n_1) = 8916$

$E(n_2) = 8439$

TABLE 2: ESTIMATES OF UNEMPLOYMENT RATIOS (PERCENT)

	TRUE	IDENTICALS	TOTAL	SINGLES	ADJUSTED
R1	4.17	4.10	4.35	9.15	4.35
R2	4.17	3.91	3.91		4.25
R1 - R2	0.00	0.19	0.44		0.10

EXAMPLE F

First Stage Response Probabilities

$P_u = 0.93 \qquad P_e = 0.89$

Second Stage Response Probabilities

$P_{uu} = 0.50 \; P_{ue} = 0.96 \; P_{eu} = 0.50 \; P_{ee} = 0.95$

$Q_{uu} = 0.00 \; Q_{ue} = 0.00 \; Q_{eu} = 0.00 \; Q_{ee} = 0.00$

True Population Figures

$N_{uu} = 300 \; N_{ue} = 100 \; N_{eu} = 100 \; N_{ee} = 9500$

TABLE 1: EXPECTED SAMPLE NUMBERS

	U2	E2	N2	T2
U1	139	89	143	372
E1	44	8032	467	8544
N1	0	0		
T1	184	8122		

$$E(n_1) = 8916$$

$$E(n_2) = 8306$$

TABLE 2: ESTIMATES OF UNEMPLOYMENT RATIOS (PERCENT)

	TRUE	IDENTICALS	TOTAL	SINGLES	ADJUSTED
R1	4.17	2.83	4.35	30.65	4.35
R2	4.17	2.27	2.27		4.29
R1 - R2	0.00	0.57	2.09		0.06